3.3 Key Features of Graphs Notes

VOCABULARY

The x-**intercept** is where a graph crosses or touches the x-axis. It is the ordered pair $(a,0)$. Where a is a real number.

The y -**intercept** is where a graph crosses or touches the y-axis. It is the ordered pair $(0,b)$. Where b is a real number.

A **relative maximum** occurs when the y-value is greater than all of the y-values near it. A function may have more than one relative maximum value. A **relative minimum** occurs when the y-value is less than all of the y-values near it. A function may have more than one relative minimum value.

An **interval** is a set of numbers between two x-values. An **open interval** is a set of numbers between two x-values that does not include the two end values. **Open intervals** are written in the form (x_1, x_2) or $x_1 < x < x_2$. A **closed interval** is a set of numbers between two x-values that does include the two end values. **Closed intervals** are written in the form $[x_1, x_2]$ or $x_1 \le x \le x_2$.

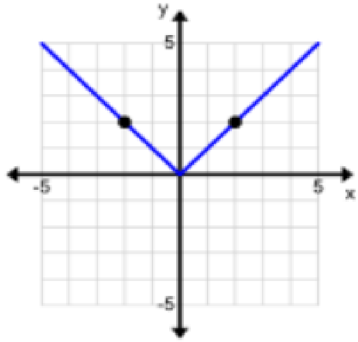
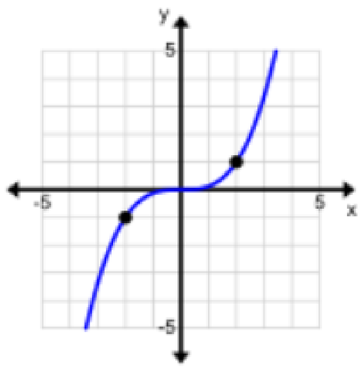
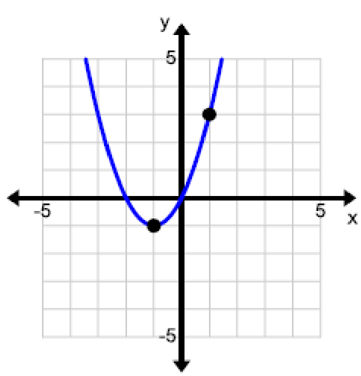
A function f is **increasing** when it is rising (or going up) from left to right and it is **decreasing** when it is falling (or going down) from left to right. A **constant** function is neither increasing nor decreasing; it has the same y-value for its entire domain.

A function is **positive** when $f(x) > 0$ or the y-coordinates are always positive. A function is **negative** when $f(x) < 0$ or the y-coordinates are always negative.

End behavior describes what is happening to the y-values of a graph when x goes to the far right $(+\infty)$ or x goes the far left $(-\infty)$.

End behavior is written in the following format:

Right End Behavior:	**Left End Behavior:**
$\lim_{x \to \infty} f(x) = c$	$\lim_{x \to -\infty} f(x) = c$

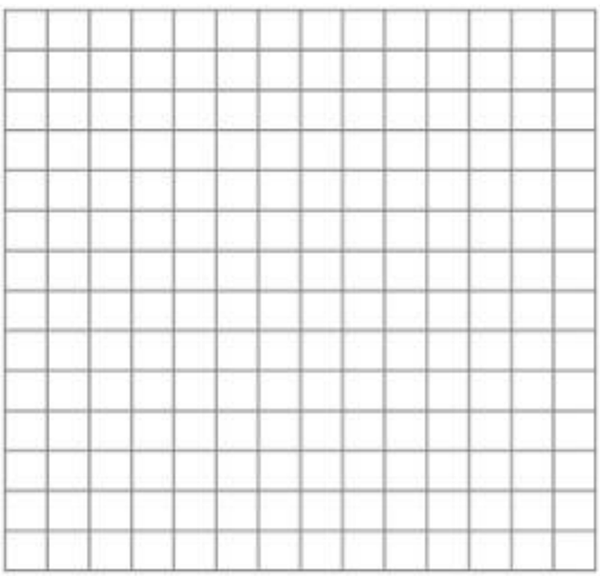
VOCABULARY	GRAPHICALLY	ALGEBRAICALLY
A function is **symmetric with respect to the y-axis** if, for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph. In other words, if you substitute $-x$ in for every x you end up with the original function. When looking at the graph, you could "fold" the graph along the y-axis and both sides are the same.		$f(x)=\|x\|+5$ $f(-x)=\|-x\|+5$ $f(x)=f(-x)=\|x\|+5$
A function is **symmetric with respect to the origin** if, for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph. In other words, if you substitute $-x$ in for every x you end up with the opposite of the original function. When looking at the graph, there is a mirror image in Quadrants 1 & 3 or Quadrants 2 & 4.		$f(x)=8x^3$ $f(-x)=8(-x)^3$ $f(-x)=-f(x)=-8x^3$
An equation with **no symmetry**. If you substitute $-x$ in for every x you end up with something that is neither the original function nor its opposite. When looking at the graph, you could not "fold" the graph along the y-axis and have both sides the same. It also does not reflect a mirror image in opposite quadrants.		$f(x)=x^2+2x$ $f(-x)=(-x)^2+2(-x)$ $f(-x)=x^2-2x\neq f(x)\neq -f(x)$

Examples:

Analyze the key features of $f(x)$.

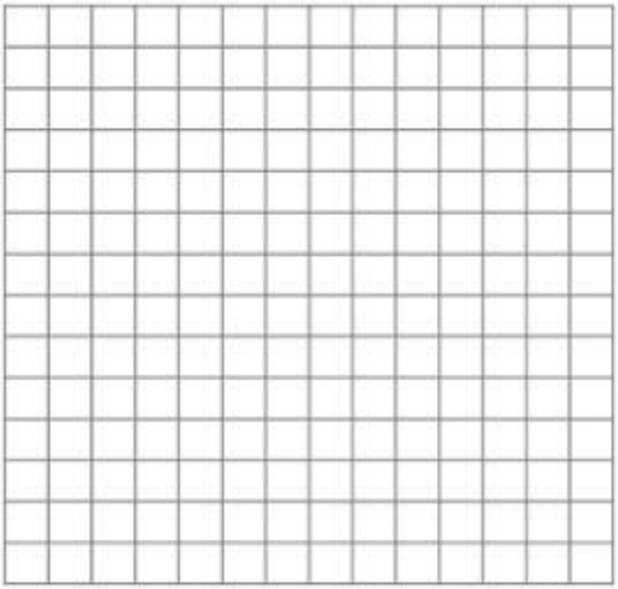
a. Graph the function
b. Identify the intercepts
c. Identify the relative maximums and minimums
d. Identify the intervals where the function is increasing or decreasing
e. Identify the intervals where the function is positive or negative
f. Determine the end behavior
g. Determine the symmetry

1. $f(x) = 2x + 3$



2. $f(x) = -x^3 + 2x$

3. $f(x) = 2\sqrt{x - 1}$



4. $f(x) = e^{x+2} + 1$