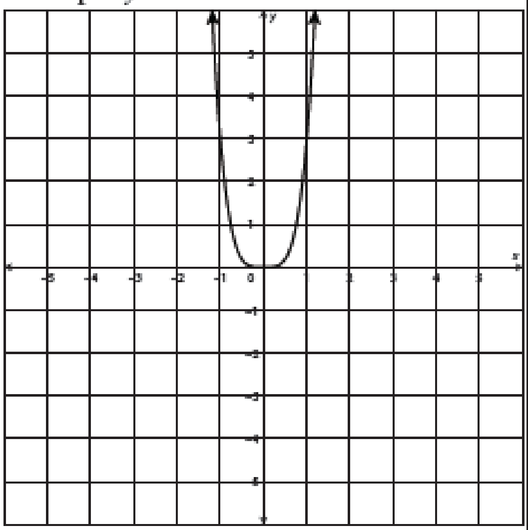
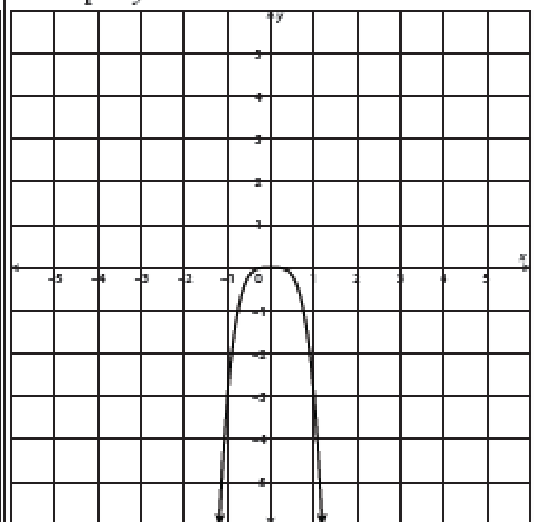
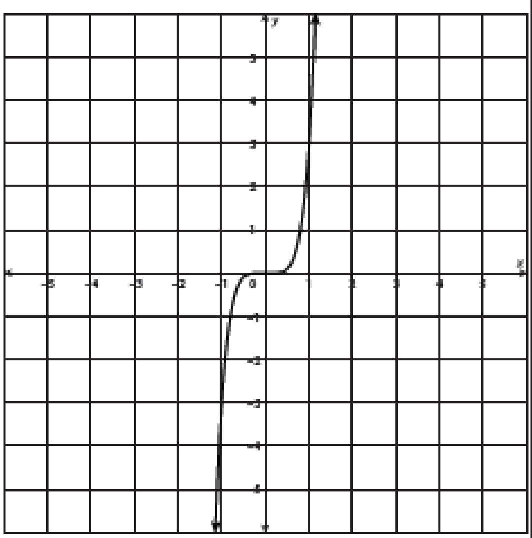
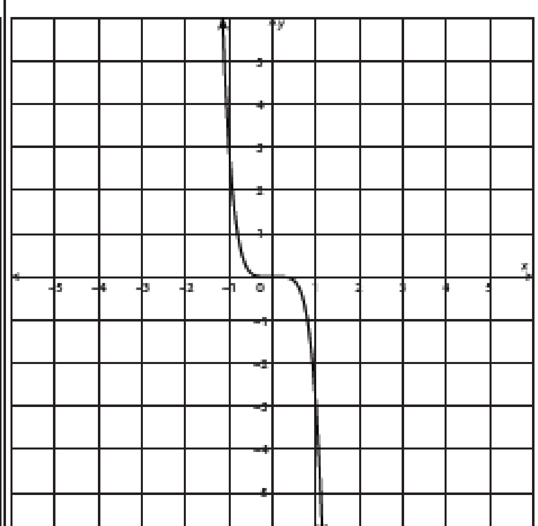
1.6 Graphing Polynomials Notes

End Behavior

- To determine the **end behavior** of a polynomial function, or the behavior of the graph as x approaches positive or negative infinity, consider the highest degree of the polynomial and its coefficient, ax^n.

Even-degree polynomials	
Positive leading coefficient Example: $y = 3x^4$	**Negative leading coefficient** Example: $y = -3x^4$
	
$f(x) \to +\infty$ as $x \to -\infty$ $f(x) \to +\infty$ as $x \to +\infty$	$f(x) \to -\infty$ as $x \to -\infty$ $f(x) \to -\infty$ as $x \to +\infty$

Odd-degree polynomials	
Positive leading coefficient Example: $y = 3x^5$	**Negative leading coefficient** Example: $y = -3x^5$
	
$f(x) \to -\infty$ as $x \to -\infty$ $f(x) \to +\infty$ as $x \to +\infty$	$f(x) \to +\infty$ as $x \to -\infty$ $f(x) \to -\infty$ as $x \to +\infty$

Turning Points

- A **turning point** of a function is a point where the graph of the function changes from sloping upward to sloping downward or, alternatively, from sloping downward to sloping upward.
- To determine the maximum number of turning points of a function, subtract 1 from the highest degree of the polynomial. In other words, find $n - 1$.
- For instance, the polynomial function $y = 3x^7 + 9x^3 - x + 4$ can have no more than $7 - 1$, or 6, turning points.
- The maximum number of turning points does not necessarily indicate the actual number of turning points of a function, just that it can have no more than that number. Some functions may have fewer turning points than the number calculated.
- A turning point corresponds to a **local maximum**, the greatest value of a function for a particular interval of the function, or a **local minimum**, the least value of a function for a particular interval of the function. A local maximum may also be referred to as a **relative maximum** and a local minimum may also be referred to as a **relative minimum**.

Roots of a Polynomial Function

- The highest degree of the polynomial determines the maximum number of **roots**, or x-intercepts of a function.
- A polynomial function with a degree of 10 could have up to 10 roots, but could also have 0 to 9 roots, depending on the specific equation.
- Recall that real numbers include all rational and irrational numbers, but do not include imaginary and complex numbers.

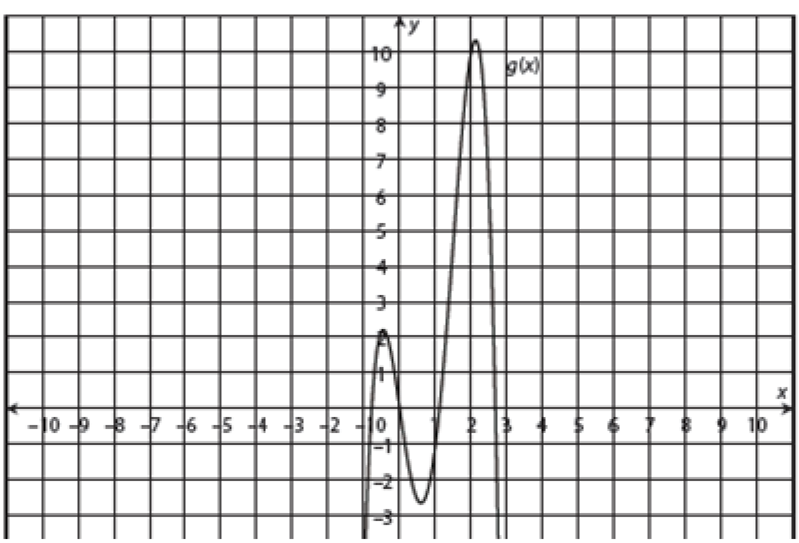
Guided Practice 2A.3.1

Example 1

Determine the end behavior, maximum number of turning points, and maximum number of real roots of the function $f(x) = 6x^5 - 3x^4 + 2x + 7$.

Example 2

Describe the end behavior of the given graph of $g(x)$. Determine whether the graph represents an even-degree or odd-degree function, and determine the number of real roots.



Example 3

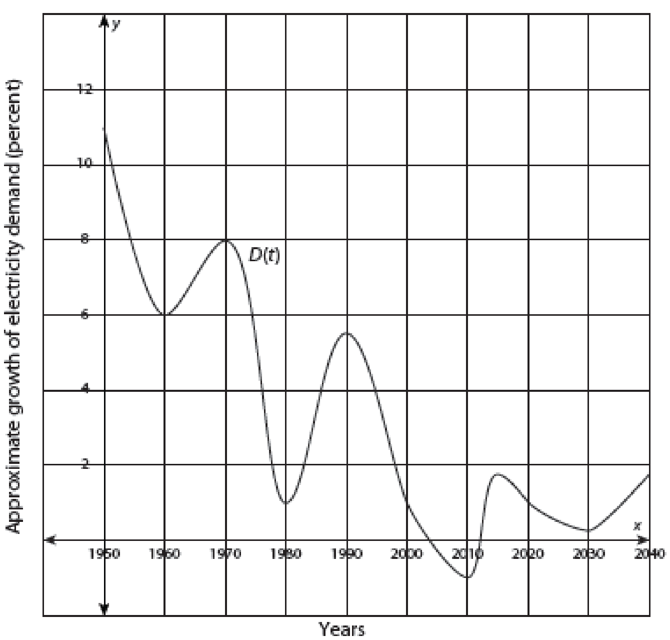
Use a graphing calculator to graph the function $p(x) = -x^4 + 3x^2 + 4$. Summarize the end behavior and turning points of the function.

Example 4

Create a rough sketch of the graph of a sixth-degree polynomial function with a positive leading coefficient.

Problem-Based Task 2A.3.1: It's Electric!

The growth of demand for electricity in the United States changes from year to year. Below is a sketch of a polynomial function that represents that approximate growth since 1950 and includes projections to 2040. Consider a possible polynomial function that could represent this information. Using end behavior, turning points, and roots, do you expect the growth of electricity demand to increase or decrease from 2040 to 2050? Explain your reasoning.



Source: U.S. Energy Information Administration: Annual Energy Outlook 2013

Finding Zeros, Multiplicity, and Graphing

Multiplicity of Zeros: Recall that the Fundamental Theorem of Algebra states that zeros can be repeated. When a zero is repeated, the same factor occurs multiple times. We say the factor has a multiplicity of the number of times it is repeated. For instance, $(x-5)^4$ means that the zero $(5,0)$ is repeated four times and has a multiplicity of 4.

NOTE: When the multiplicity of a zero is even, the graph of the function touches the x-axis. When the multiplicity of a zero is odd, the graph of the function crosses the x-axis.

Guided Practice 2A.3.3

Example 1

In each polynomial, identify the zeros, their multiplicity, and determine whether they touch or cross the x-axis at each zero.

1. $f(x) = (x-4)^2(x-1)^3$

Zero	Multiplicity	Touch/Cross

2. $f(x) = (x-1)^2(x+2)$

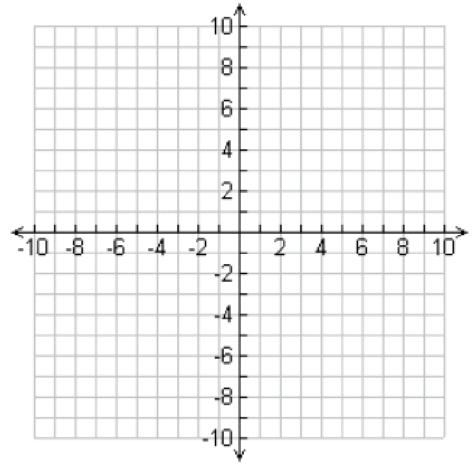
Zero	Multiplicity	Touch/Cross

To graph a polynomial, we need to know the following:

a. **End behavior of the graph**
b. **Zeros of the graph**
c. **Multiplicity of zeros (tells us if the graph crosses or touches each zero)**

Draw a rough sketch of the graph of each polynomial. Use technology to check your answer.

3. $f(x) = (x+3)^2(x^2-9)$

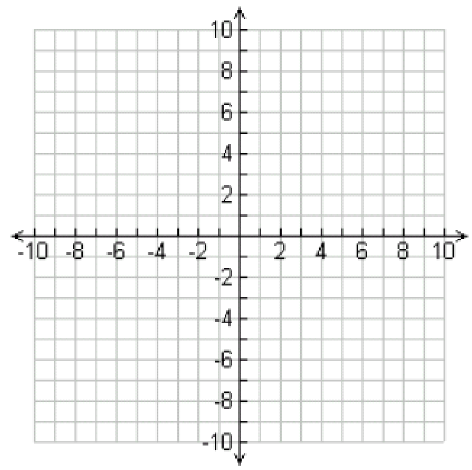


a. End behavior:

b. Zeros:

c. Multiplicity:

4. $f(x) = -x(x+5)^4\,(x-2)^3$



a. End behavior:

b. Zeros:

c. Multiplicity: