10.3 - Trigonometric Functions and Their Inverses

In other sections, you learned that for a function $f(f^{-1}(x))=x$ for all values of x for which $f^{-1}(x)$ is defined. If this property is applied to the trigonometric functions, the following equations will be true whenever they are defined:

$$\sin(\sin^{-1}(x))=x \qquad \cos(\cos^{-1}(x))=x \qquad \tan(\tan^{-1}(x))=x$$

As well, you learned that $f^{-1}(f(x))=x$ for all values of x for which $f(x)$ is defined. If this property is applied to the trigonometric functions, the following equations that deal with finding an inverse trig function of a trig function, will only be true for values of x within the restricted domains.

$$\sin^{-1}(\sin(x))=x \qquad \cos^{-1}(\cos(x))=x \qquad \tan^{-1}(\tan(x))=x$$

Examples:

1. $\sin(\sin^{-1}(.75)) =$

2. $\cos^{-1}(\cos(\frac{3\pi}{4})) =$

3. $\tan(\tan^{-1}(1)) =$

Composite Trigonometric Functions Notes

Term	Definition
composite function	A composite function is a function $h(x)$ formed by using the output of one function $g(x)$ as the input of another function $f(x)$. Composite functions are written in the form $h(x) = f(g(x))$ or $h = f \circ g$.

It is possible to have **composite functions** that are composed of one trigonometric function in conjunction with another different trigonometric function.

When solving these types of problems, **start with the function that is composed inside of the other and work your way out**. Use the following problems as a guideline.

1. Find $\sin(\sin^{-1}(\frac{\sqrt{2}}{2}))$

2. Without using technology, find the exact value of each of the following:

   a. $\cos(\tan^{-1}(\sqrt{3}))$

   b. $\tan\left(\sin^{-1}\left(\frac{-1}{2}\right)\right)$

   c. $\cos(\tan^{-1}(-1))$

   d. $\sin^{-1}\left(\cos\left(\frac{3\pi}{2}\right)\right)$

Inverse & Composite Trig Equations Continued

For some cases, you will be evaluating at a number or equation not on the unit circle. In this scenario, draw a triangle and use SohCahToa to help you solve.

1. Evaluate $\sin\left(\cos^{-1}\frac{5}{13}\right)$

2. Evaluate $\tan\left(\sin^{-1}\frac{6}{11}\right)$

3. Evaluate $\cos\left(\sin^{-1}\left(\frac{1}{x}\right)\right)$

4. Evaluate $\sin\left(\cos^{-1}\left(\frac{\sqrt{x^2-9}}{x}\right)\right)$