10.4 - Trigonometric Identities Notes

Proving an identity is very different in concept from solving an equation. Though you'll use many of the same techniques, they are *not* the same, and the differences are what can cause you problems.

An "identity" is an equation or statement that is always true, no matter what. For instance, $sin(x) = 1/csc(x)$ is an identity. To "prove" an identity, you have to **use logical steps to show that one side of the equation can be transformed into the other side of the equation**. You do *not* plug values into the identity to "prove" anything. There are infinitely-many values you can plug in. Are you really going to "prove" anything by listing three or four values where the two sides of the equation are equal? Of course not. And sometimes you'll be given an equation which is *not* an identity. If you plug a value in where the two sides happen to be equal, such as $\pi/4$ for the (false) identity $sin(x) = cos(x)$, you could fool yourself into thinking that a mere equation is an identity.

*** * To prove an identity, you cannot work on both sides of the equation at the same time. * ***

You can work on both sides together for a regular equation, because you're trying to find where the equation is true. When you are working with an identity, if you work on both sides and work down to where the sides are equal, you will only have shown that, *if* the starting equation is true, then you can arrive at another true equation. But you won't have proved, logically, that the original equation was actually true.

Since you'll be working with two sides of an equation, it might be helpful to introduce some notation. The "left-hand side" of an equation is denoted by **LHS**, and the "right-hand side" is denoted as **RHS**.

Some helpful Trigonometric Identities to obtain equivalent forms:

Quotient Identities	Reciprocal Identities	Pythagorean Identities
$\tan\theta = \frac{\sin\theta}{\cos\theta},\ \cos\theta \neq 0$	$\csc\theta = \frac{1}{\sin\theta},\ \sin\theta \neq 0$	$\sin^2\theta + \cos^2\theta = 1$
$\cot\theta = \frac{\cos\theta}{\sin\theta},\ \sin\theta \neq 0$	$\sec\theta = \frac{1}{\cos\theta},\ \cos\theta \neq 0$	$1 + \tan^2\theta = \sec^2\theta$
	$\cot\theta = \frac{1}{\tan\theta},\ \tan\theta \neq 0$	$1 + \cot^2\theta = \csc^2\theta$

1. **Prove the identity:** $\frac{\cot(x)}{\csc(x)} = \cos(x)$

2. **Prove the identity:** $\cot(x) + \tan(x) = \sec(x)\csc(x)$