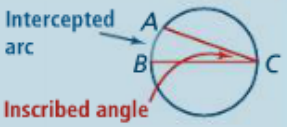
12.4: Inscribed Angles Notes

An angle whose vertex is on the circle and whose sides are chords of the circle is an **inscribed angle**. An arc with endpoints on the sides of an inscribed angle, and its other points in the interior of the angle, is an **intercepted arc**. In the diagram, inscribed $\angle C$ intercepts $\overset{\frown}{AB}$.



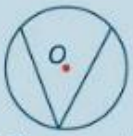
take note

Theorem 84 Inscribed Angle Theorem

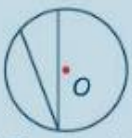
The measure of an inscribed angle is half the measure of its intercepted arc.

$$m\angle B = \frac{1}{2} m\overset{\frown}{AC}$$

I: The center is on a side of the angle.



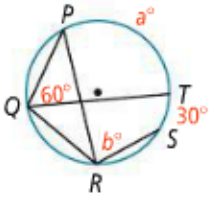
II: The center is inside the angle.



III: The center is outside the angle.

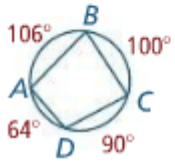
Problem 1 Using the Inscribed Angle Theorem

What are the values of a and b?



a. In $\odot O$, what is $m\angle A$?

b. What are $m\angle A$, $m\angle B$, $m\angle C$, and $m\angle D$?

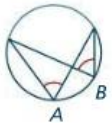


take note

Corollaries to Theorem 84 The Inscribed Angle Theorem

Corollary 1

Two inscribed angles that intercept the same arc are congruent.

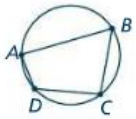


Corollary 2

An angle inscribed in a semicircle is a right angle.

Corollary 3

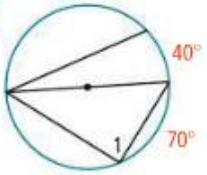
The opposite angles of a quadrilateral inscribed in a circle are supplementary.



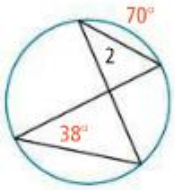
Problem 2 Using Corollaries to Find Angle Measures

What is the measure of each numbered angle?

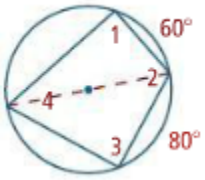
A



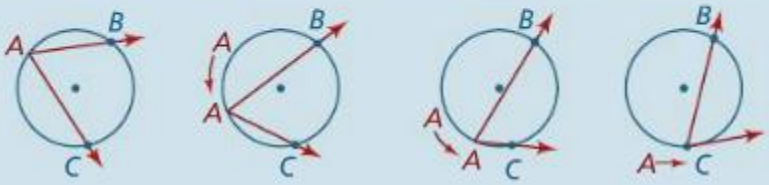
B



Got It? In the diagram at the right, what is the measure of each numbered angle?



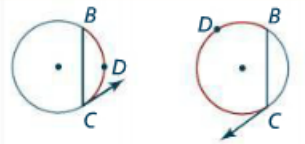
The following diagram shows point A moving along the circle until a tangent is formed. From the Inscribed Angle Theorem, you know that in the first three diagrams $m\angle A$ is $\frac{1}{2}m\,\overset{\frown}{BC}$. As the last diagram suggests, this is also true when A and C coincide.



take note

Theorem 85

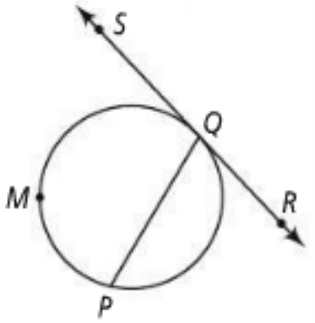
The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc.



$m\angle C = \frac{1}{2} m\overset{\frown}{BDC}$

Problem 3 Using Arc Measure

In the diagram, $\overleftrightarrow{SR}$ is a tangent to the circle at Q. If $m\overset{\frown}{PMQ} = 212$, what is $m\angle PQR$?



Got It? a. In the diagram at the right, $\overline{KJ}$ is tangent to $\odot O$. What are the values of x and y?