2.2

Multiplying and Dividing Rational Expressions



Many problems require finding products and quotients of rational expressions.

Essential Understanding You can multiply and divide rational expressions using the same properties you use to multiply and divide numerical fractions.

If a, b, c, and d represent polynomials (where $b \neq 0$ and $d \neq 0$), then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.

Problem 1 Multiplying Rational Expressions

Got It? What is the product? State any excluded values.

$$\mathbf{a}.\ \frac{5}{y}\cdot\frac{3}{y^3}$$

b.
$$\frac{x}{x-2} \cdot \frac{x+1}{x-3}$$

A Practice Multiply. State any excluded values.

1.
$$\frac{5}{3a^2} \cdot \frac{8}{a^3}$$

2.
$$\frac{m-2}{m+2} \cdot \frac{m}{m-1}$$

Problem 2 Using Factoring

Got lt? a. What is the product $\frac{3x^2}{x+2} \cdot \frac{x^2+3x+2}{x}$?



3.
$$\frac{t^2-t-12}{t+1} \cdot \frac{t+1}{t+3}$$

4.
$$\frac{4x+1}{5x+10} \cdot \frac{30x+60}{2x-2}$$

You can also multiply a rational expression by a polynomial. Leave the product in factored form.

Problem

Multiplying a Rational Expression by a Polynomial

Got It? What is the product?

a.
$$\frac{2x-14}{4x-6} \cdot (6x^2-13x+6)$$

b.
$$\frac{x^2+2x+1}{x^2-1}$$
 • (x^2+2x-3)

A Practice Multiply.

5.
$$\frac{h-1}{6h+3} \cdot (2h^2+9h+4)$$

6.
$$(w^2 - 8w + 15) \cdot \frac{w+3}{4w-20}$$

Recall that $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$, where $b \neq 0$, $c \neq 0$, and $d \neq 0$. When you divide rational expressions, first rewrite the quotient as a product, using the reciprocal, before dividing out common factors.

Problem

Problem 4 Dividing Rational Expressions

Got It? What is the quotient?

a.
$$\frac{x}{x+y} \div \frac{xy}{x+y}$$

b.
$$\frac{4k+8}{6k-10} \div \frac{k^2+6k+8}{9k-15}$$

7.
$$\frac{x-1}{x+4} \div \frac{x+3}{x+4}$$

8.
$$\frac{x^2+6x+8}{x^2+x-2} \div \frac{x+4}{2x+4}$$

The reciprocal of a polynomial such as $x^2 + 3x + 2$ is $\frac{1}{x^2 + 3x + 2}$.

Problem 5

Problem 5 Dividing a Rational Expression by a Polynomial

Got lt? What is the quotient $\frac{z^2-2z+1}{z^2+2} \div (z-1)$?

9.
$$\frac{11k+121}{7k-15}$$
 ÷ $(k+11)$

10.
$$\frac{x^2+10x-11}{x^2+12x+11} \div (x-1)$$

A complex fraction is a fraction that contains one or more fractions in its numerator, in its denominator, or in both. You can simplify a complex fraction by dividing its numerator by its denominator.

Any complex fraction of the form $\frac{\frac{a}{b}}{\frac{c}{d}}$ (where $b \neq 0$, $c \neq 0$, and $d \neq 0$) can be expressed as $\frac{a}{b} \div \frac{c}{d}$.



Problem 6 Simplifying a Complex Fraction

Got lt? What is the simplified form of $\frac{\frac{1}{q+4}}{\frac{2q^2}{2q+8}}$?

11.
$$\frac{\frac{g+2}{3g-1}}{\frac{g^2+2g}{6g+2}}$$

12.
$$\frac{\frac{c+4}{c^2+5c+6}}{\frac{3c^2+12c}{2c^2+5c-3}}$$