

2.5

Solving Square Root and Other Radical Equations

A **radical equation** is an equation that has a variable in a radicand or a variable with a rational exponent. If the radical has index 2, the equation is a **square root equation**. In this lesson, assume that all radicals and expressions with rational exponents represent real numbers.

Essential Understanding Solving a square root equation may require that you square each side of the equation. This can introduce extraneous solutions.

To solve a radical equation, isolate the radical on one side of the equation. Then raise each side to the power suggested by the index.


Problem 1 Solving a Square Root Equation

Got It? What is the solution of $\sqrt{4x + 1} - 5 = 0$?

A Practice Solve.

1. $3\sqrt{x + 3} = 15$

2. $\sqrt{2x + 3} - 7 = 0$

To solve equations of the form $\left(x^{\frac{m}{n}}\right) = k$, raise each side of the equation to the power $\frac{n}{m}$, the reciprocal of $\frac{m}{n}$. If either m or n is even, then $\left(x^{\frac{m}{n}}\right)^{\frac{n}{m}} = |x|$.


Problem 2 Solving Other Radical Equations

Got It? What are the solution(s) of $2(x + 3)^{\frac{2}{3}} = 8$?

A Practice Solve.

3. $3(x - 2)^4 = 24$

4. $(x + 1)^2 - 2 = 25$



Problem 3 Using Radical Equations

Got It? A crater similar to the one in Problem 3 has diameter 1 km.
STEM What is the volume of the crater? Use the formula $d = 2\sqrt[3]{\frac{V}{0.3}}$, which relates the diameter d of the rim (in meters) to the volume V (in cubic meters).



Practice 5. **Volume** A spherical water tank holds 9000 ft³ of water. What is the diameter of the tank? (*Hint:* $\frac{1}{6}d^3\pi = V$)

STEM 6. **Hydraulics** The formula $\frac{\pi d^2 v}{4} = Q$ models the diameter of a pipe where Q is the maximum flow of water in a pipe, and v is the velocity of the water. What is the diameter of a pipe that allows a maximum flow of 30 ft³/min of water flowing at a velocity of 400 ft/min? Round your answer to the nearest inch.

When you raise each side of an equation to a power, it is possible to introduce extraneous solutions. Therefore, it becomes very important that you check all solutions in the original equation. A correct solution will give a true statement. An extraneous solution will give a false statement.



Problem 4 Checking for Extraneous Solutions

Got It? a. What is the solution of $\sqrt{5x - 1} + 3 = x$? Check your results.



Practice Solve. Check for extraneous solutions.

7. $\sqrt{-3x - 5} = x + 3$

8. $(x + 3)^{\frac{1}{2}} - 1 = x$



Problem 5 Solving an Equation With Two Radicals

Got It? What is the solution of $\sqrt{5x + 4} - \sqrt{x} = 4$?

A Practice Solve. Check for extraneous solutions.

9. $(2x)^{\frac{1}{2}} = (x + 5)^{\frac{1}{2}}$

10. $\sqrt{3 - x} + \sqrt{x + 2} = 3$