

4.4 Inverse Relations

If a relation pairs element a of its domain to element b of its range, the **inverse relation** pairs b with a . So if (a, b) is an ordered pair of a relation, then (b, a) is an ordered pair of its inverse. If both a relation and its inverse happen to be functions, they are **inverse functions**.

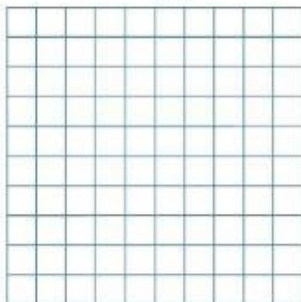
Essential Understanding The inverse of a function may or may not be a function.

This diagram shows a relation r (a function) and its inverse (not a function). The range of the relation is the domain of the inverse. The domain of the relation is the range of the inverse.



Problem 1 Finding the Inverse of a Relation

Got It? a. What are the graphs of t and its inverse?



Relation t

x	0	1	2	3
y	-5	-4	-3	-3

Reasoning b. Is t a function? Is the inverse of t a function? Explain.

As shown in Problem 1, the graphs of a relation and its inverse are the reflections of each other in the line $y = x$. If you describe a relation or function by an equation in x and y , you can switch x and y to get an equation for the inverse.



Problem 2 Finding an Equation for the Inverse

Got It? What is the inverse of $y = 2x + 8$?

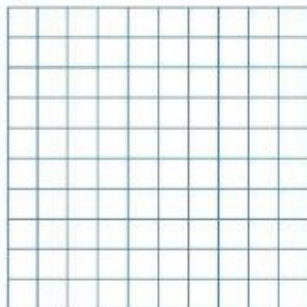
A Practice Find the inverse of each function. Is the inverse a function?

3. $y = 4 - 3x$

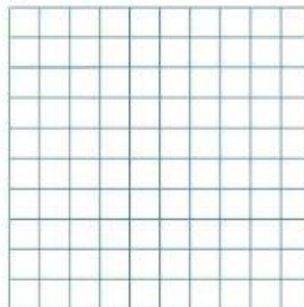
4. $y = (1 - 2x)^2 + 5$

A Practice Graph each relation and its inverse.

5. $y = 3 - 7x$



6. $y = (3 - 2x)^2 - 1$



The inverse of a function f is denoted by f^{-1} . You read f^{-1} as “the inverse of f ” or as “ f inverse.” The notation $f(x)$ is used for functions, but the relation f^{-1} may not even be a function.



Problem 4 Finding an Inverse Function

Got It? Let $g(x) = 6 - 4x$.

a. What are the domain and range of g ?

b. What is the inverse of g ?

c. What are the domain and range of g^{-1} ?



Practice For each function, find the inverse and the domain and range of the function and its inverse. Determine whether the inverse is a function.

7. $f(x) = \sqrt{x+7}$

8. $f(x) = 2x^2 + 2$



Problem 5 Finding the Inverse of a Formula

Got It? The function $d = \frac{v^2}{19.6}$ relates the distance d , in meters, that an object has fallen to its velocity v , in meters per second. Find the inverse of this function. What is the velocity of the cliff diver from Problem 5 in meters per second as he enters the water?



Practice 9. **Temperature** The formula for converting temperatures from Celsius to Fahrenheit is $F = \frac{9}{5}C + 32$.

a. Find the inverse of the formula. Is the inverse a function?

b. Use the inverse to find the Celsius temperature that corresponds to 25°F.

10. **Geometry** The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

a. Find the inverse of the formula. Is the inverse a function?

b. Use the inverse to find the radius of a sphere that has a volume of 35,000 ft³.