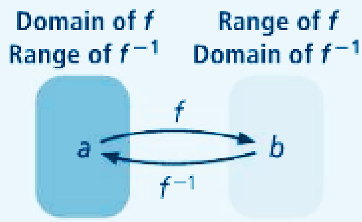
4.5 Verifying Inverse by Composition of Functions

You know that for any function f, each x-value in the domain corresponds to exactly one y-value in the range. For a **one-to-one function**, it is also true that each y-value in the range corresponds to exactly one x-value in the domain. A one-to-one function f has an inverse f^{-1} that is also a function. If f maps a to b, then f^{-1} must map b to a.



take note

Key Concept Composition of Inverse Functions

Functions f and g are inverse functions if and only if
$(f \circ g)(x) = x$ and $(g \circ f)(x) = x$ for x in the domain of g and f, respectively.

This says that the composition of a function and its inverse is essentially the identity function, $id(x) = x$, or $y = x$.

Problem 6 Composing Inverse Functions

Got It? Let $g(x) = \frac{4}{x+2}$. What is each of the following?

a. $g^{-1}(x)$ **b.** $(g \circ g^{-1})(0)$ **c.** $(g^{-1} \circ g)(0)$

To verify two functions are inverses, use composition of functions

Composition of Inverse Functions
If f and f^{-1} are inverse functions, then $\left(f^{-1} \circ f\right)(x) = x$ and $\left(f \circ f^{-1}\right)(x) = x$ for all x in the domains of f and f^{-1} respectively.

So to verify f(x) and g(x) are inverses of each other, show that (g ᵒ f)(x)=x and (f ᵒ g)(x)=x

***(g ᵒ f)(x)=x means g(f(x)), so you are substituting in f(x) for x in the function g(x)

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Examples:

1. Verify that $f(x) = 3x - 2 \; and \; g(x) = \frac{x+2}{3}$ are inverses of each other.

2. Verify that $f(x) = x^3 - 8 \; and \; g(x) = \sqrt[3]{x+8}$ are inverses of each other.

3. Verify that $f(x) = \sqrt{x+10} - 6 \; and \; g(x) = (x+6)^2 - 10$ are inverses of each other.

<u>To use a table to find an inverse function, switch the x and f(x) values, keep x the same and switch f(x) to $f^{-1}(x)$</u>

Example: Use the table below to find the inverse function

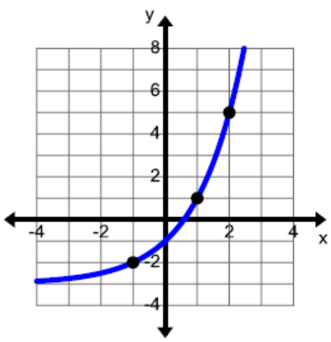
x	$f(x)=x^3-4x+1$
-7	-314
-6	-191
-5	-104
-4	-47
-3	-14
-2	1
-1	4

*****The domain of the original function becomes the range of the inverse function and the range of the original function becomes the domain of the inverse function.*****

<u>To use the graph to find the inverse function, reflect the graph over the line x=y (switch the x- and y-coordinates)</u>

Examples: Use the graphs to find the inverse functions

1.



2.