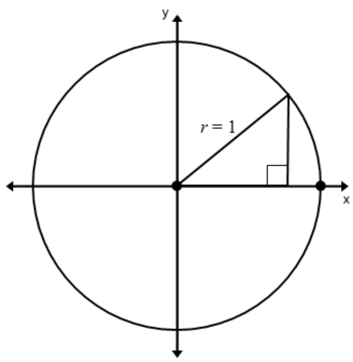
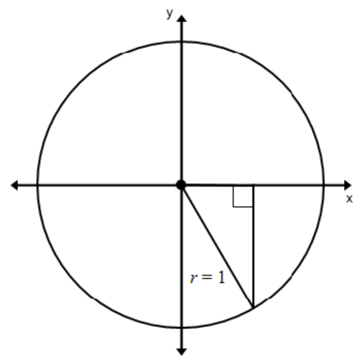
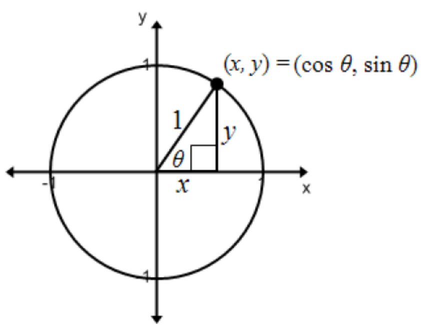
Math 3 - The Unit Circle Notes

A circle with radius 1 is called a unit circle. The unit circle provides a connection between trigonometric ratios and the trigonometric functions. We can place it on a coordinate plane and use right triangle trigonometry to find the basic trigonometric ratios. Any right triangle with hypotenuse of length 1 can be drawn in any quadrant of the unit circle.





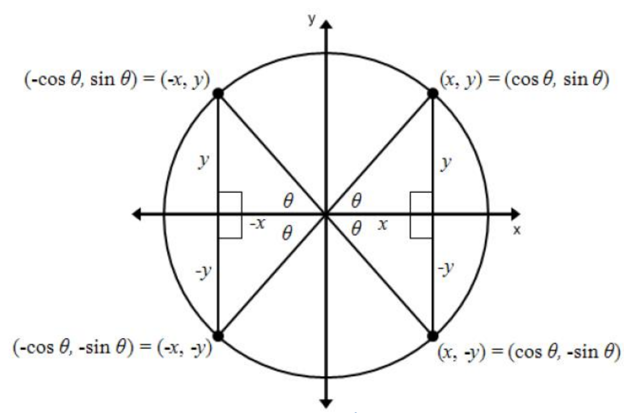
Trigonometric Ratios for a Circle of Radius 1



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\sin\theta = \frac{y}{1} \qquad \cos\theta = \frac{x}{1}$$

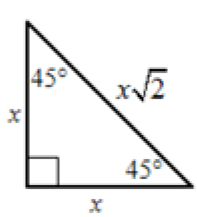
$$\sin\theta = y \qquad \cos\theta = x$$



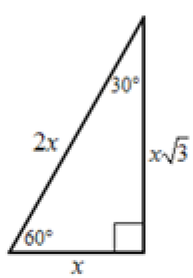
For any point (x, y) on a unit circle, the x-coordinate is the cosine of the angle and the y-coordinate is the sine of the angle. Recall that if you reflect any point (x, y) on the coordinate plane across the x axis, y axis or through the origin, then the following relationships exist:

There are special right triangles with special relationships between the lengths of their sides. These relationships can be used to simplify calculations when finding missing angles and sides.

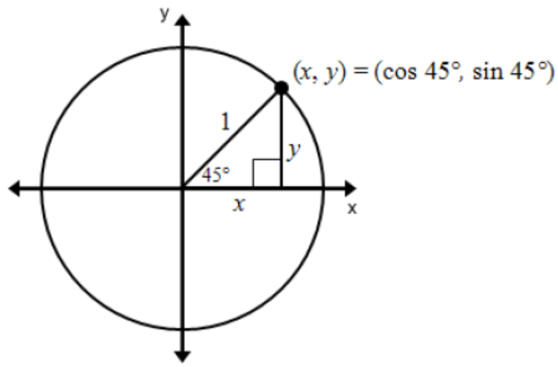
$45° - 45° - 90°$ Triangle



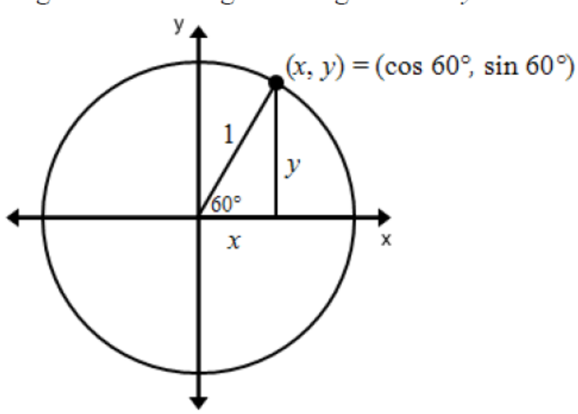
$30° - 60° - 90°$ Triangles



The figure illustrates how these triangles can be used to derive parts of the unit circle.

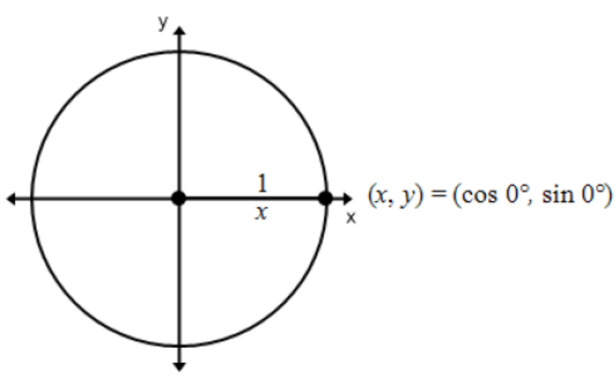


To illustrate a $30° - 60° - 90°$ triangle on the unit circle, create a right triangle with central angle $60°$. The hypotenuse is length 1 and the legs are lengths x and y.

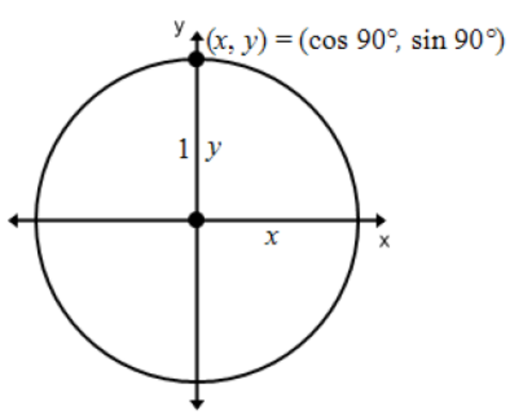


We can also consider the case where $\theta = 30°$ and use the $30° - 60° - 90°$ triangle to find the values of x and y for this value of θ.

If we plot all of the points where $\theta = 30°$, $45°$, and $60°$ and their reflections, then we get most of the unit circle. To obtain the rest of the unit circle we have to examine what happens to a point when $\theta = 0°$.



Finally we need to observe what happens when we rotate a point $90°$ from the positive x-axis.



Plotting all of the points, we obtain what is referred to as the unit circle.

The unit circle can be used to find exact values of trigonometric ratios for the angles that relate to the special right triangle angles.

Example 1:

Use the unit circle to find the exact value.

a. $\sin 135°$

b. $\cos\frac{5\pi}{4}$

Defining Tangent Values

Another way to write $\tan\theta$ is $\tan\theta = \frac{\sin\theta}{\cos\theta}$.

Example 2:

Find $\tan\frac{7\pi}{6}$

Tangent Values for the Angles on the Unit Circle

θ	0°	30°	45°	60°	90°	120°	135°	150°
θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$
θ	180°	210°	225°	240°	270°	300°	315°	330°
θ	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$