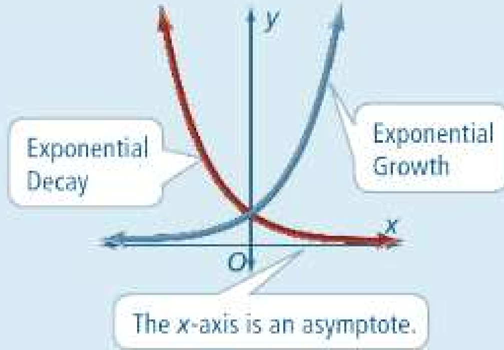
7.4: *Applications of Logarithms Notes*



Two types of exponential behavior are *exponential growth* and *exponential decay*.

For **exponential growth**, as the value of x increases, the value of y increases. For **exponential decay**, as the value of x increases, the value of y decreases, approaching zero.

The exponential functions shown here are *asymptotic* to the x-axis. An **asymptote** is a line that a graph approaches as x or y increases in absolute value.

take note

Concept Summary Exponential Functions

For the function $y = ab^x$,

- if $a > 0$ and $b > 1$, the function represents exponential growth.
- if $a > 0$ and $0 < b < 1$, the function represents exponential decay.

In either case, the y-intercept is $(0, a)$, the domain is all real numbers, the asymptote is $y = 0$, and the range is $y > 0$.

For exponential growth $y = ab^x$, with $b > 1$, the value b is the **growth factor.** A quantity that exhibits exponential growth increases by a constant percentage each time period. The percentage increase r, written as a decimal, is the *rate of increase* or *growth rate*. For exponential growth, $b = 1 + r$.

For exponential decay, $0 < b < 1$, and b is the **decay factor**. The quantity decreases by a constant percentage each time period. The percentage decrease, r, is the *rate of decay*. Usually a rate of decay is expressed as a negative quantity, so $b = 1 + r$.

take note

Key Concept Exponential Growth and Decay

You can model exponential growth or decay with this function.

$$A(t) = a(1 + r)^t$$

- $A(t)$: Amount after t time periods
- r: Rate of growth ($r > 0$) or decay ($r < 0$)
- a: Initial amount
- t: Number of time periods

For growth or decay to be exponential, a quantity changes by a fixed percentage each time period.

Modeling Exponential Growth:

Example) You invested \$1000 in a savings account at the end of 6th grade. The account pays 5% annual interest. How much money will be in the account after six years?

Example) Suppose that a new employee starts working at \$6.99 per hour and receives a 4% raise each year. After time t, in years, his hourly wage is given by the equation $y = \$6.99(1.04)^t$. Find the amount of time after which he will be earning \$10.00 per hour.

Example) The amount of carbon-14 in an object is given by $y = a \cdot e^{-0.00012t}$, where a is the amount of carbon-14 originally in the object, and t is the age of the object in years. A fossil bone contains 29% of its originally carbon-14. What is the approximate age of the bone?

Using a Logarithmic Scale

A **common logarithm** is a logarithm with base 10. You can write a common logarithm $\log_{10} x$ simply as $\log x$, without showing the 10.

Many measurements of physical phenomena have such a wide range of values that the reported measurements are logarithms (exponents) of the values, not the values themselves. When you use the logarithm of a quantity instead of the quantity, you are using a **logarithmic scale**. The Richter scale is a logarithmic scale. It gives logarithmic measurements of earthquake magnitude.

Problem 3 Using a Logarithmic Scale

In December 2004, an earthquake with magnitude 9.3 on the Richter scale hit off the northwest coast of Sumatra. The diagram shows the magnitude of an earthquake that hit Sumatra in March 2005. The formula $\log \frac{I_1}{I_2} = M_1 - M_2$ compares the intensity levels of earthquakes where I is the intensity level determined by a seismograph, and M is the magnitude on a Richter scale. How many times more intense was the December earthquake than the March earthquake?

$\log \frac{I_1}{I_2} = M_1 - M_2$ Use the formula.

pH is also a Logarithmic Scale:

The pH of a substance equals $-\log\,[H^+]$, where $[H^+]$ is the concentration of hydrogen ions.

Problem 4 Using a Logarithmic Scale STEM

Chemistry The pH of a substance equals $-\log[H^+]$, where $[H^+]$ is the concentration of hydrogen ions. $[H^+_a]$ for household ammonia is 10^{-11}. $[H^+_v]$ for vinegar is 6.3×10^{-3}. What is the difference of the pH levels of ammonia and vinegar?

Think

Write the equation for pH.

Write the difference of the pH levels.

Substitute values for $[H^+_v]$ and $[H^+_a]$.

Use the Product Property of Logarithms, and simplify.

Use a calculator.

Write the answer.

Problem 2 Using a Logarithmic Scale

Think About a Plan The loudness in decibels (dB) of a sound is defined as $10 \log \frac{I}{I_0}$, where I is the intensity of the sound in watts per square meter (W/m^2). I_0, the intensity of a barely audible sound, is equal to 10^{-12} W/m^2. Town regulations require the loudness of construction work not to exceed 100 dB. Suppose a construction team is blasting rock for a roadway. One explosion has an intensity of 1.65×10^{-2} W/m^2. Is this explosion in violation of town regulations?

- Which physical value do you need to calculate to answer the question?
- What values should you use for I and I_0?