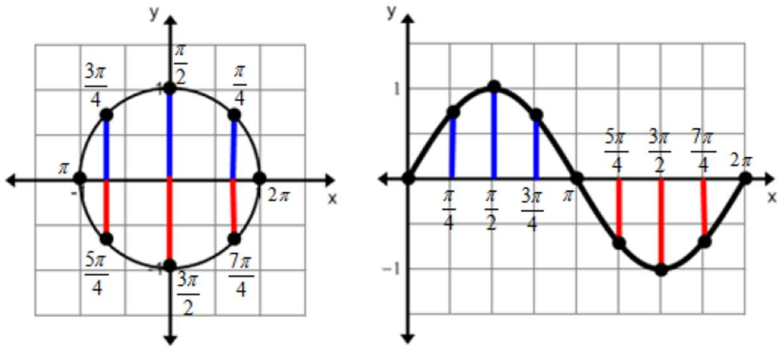
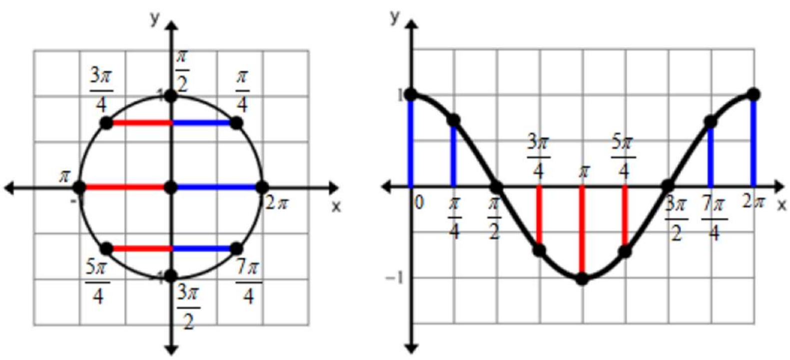
Math 3 - Graphing Sine and Cosine Notes

Graphing Sine and Cosine

Plotting every angle and its corresponding sine value, which is the y-coordinate, for different angles on the unit circle, allows us to create the sine function where $x = \theta$ and $y = \sin\theta$.



Plotting every angle and its corresponding cosine value, which is the x-coordinate, for different angles on the unit circle, allows us to create the cosine function where $x = \theta$ and $y = \cos\theta$.



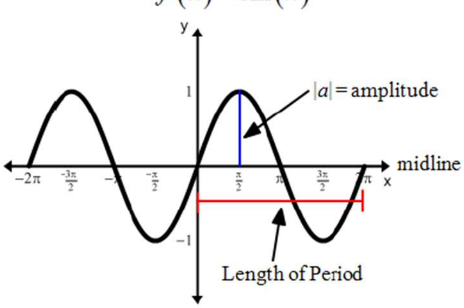
Keep in mind that we can extend this concept to all real numbers because rotating around the circle multiple times maps the new angle on to an existing coterminal angle. This type of function is called a **periodic** function because it has a pattern that repeats.

General Equations and Graphs

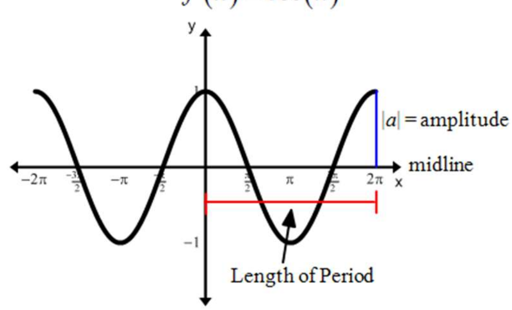
$$f(x) = a\sin(bx) + k \qquad \text{and} \qquad f(x) = a\cos(bx) + k$$

The domain of each function is the set of all real numbers, $(-\infty, \infty)$. The range of each function is $[-1, 1]$.

$f(x) = \sin(x)$



$f(x) = \cos(x)$



$$f(x) = a\sin(bx) + k \quad \text{and} \quad f(x) = a\cos(bx) + k$$

VOCABULARY

The **amplitude**, $|a|$, is half the difference between the maximum and minimum values of the function.

The **period**, $\frac{2\pi}{|b|}$, is the interval length needed to complete one cycle.

The **frequency**, $\frac{|b|}{2\pi}$, is the number of complete cycles a periodic function makes in a specific interval.

The **midline**, k, is the horizontal line that cuts the trigonometric function in half.

<u>To Graph Sine and Cosine Graphs:</u>

Step 1: Draw the midline y=k

Step 2: Divide the period $\frac{2\pi}{|b|}$ into 4 equal segments and label them along the x-axis

Step 3: Determine the maximum y=k+a, and the minimum y=k-a values and label the y-axis accordingly

Step 4: Graph the period appropriately, starting at 0

*****Pay attention to Sine vs. Cosine*****

- **From 0, sin(x) starts at midline, then goes up to maximum**
- **From 0, cos(x) starts at the maximum, then goes down**

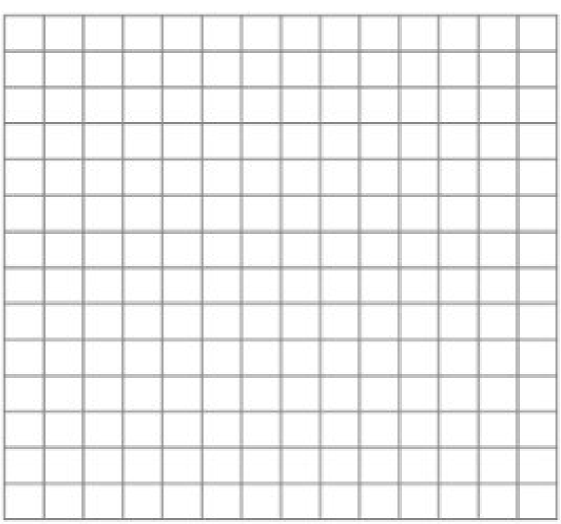
*****Pay attention to reflections*****

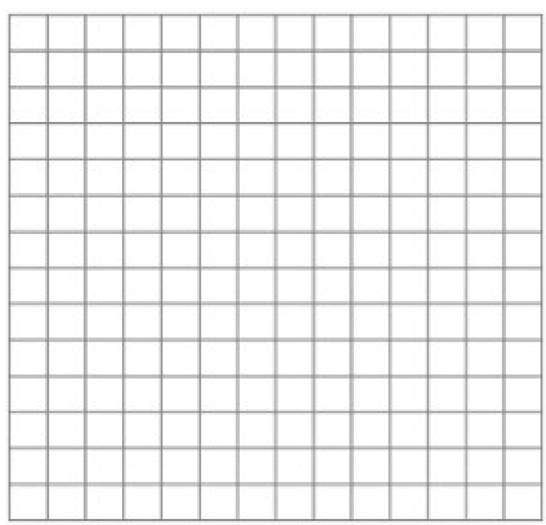
- **From 0, -sin(x) starts at midline, then goes down to minimum**
- **From 0, -cos(x) starts at minimum, then goes up**

Example 1: Identify the midline, amplitude, period, maximum, and minimum of the function, then sketch one period of the graph starting at x=0

a. $f(x) = 2\sin(3\pi x) - 1$

b. $f(x) = -5\cos\left(\frac{1}{2}x\right) + 3$





<u>Gathering midline, amplitute, period, and equation from a graph</u>

To find the midline from a graph: $k = \frac{max+min}{2}$, or find halfway between max and min

Find the amplitude $a = \frac{|max-\mathrm{m}\quad|}{2}$, or count up or down from the midline

Find period by counting how long it takes to go one full cycle (I like to start at 0 and go from there). Then use the equation $period = \frac{2\pi}{|b|}$ to solve for b

Use b, midline, and amplitute to fill in the equation from the appropriate parent function

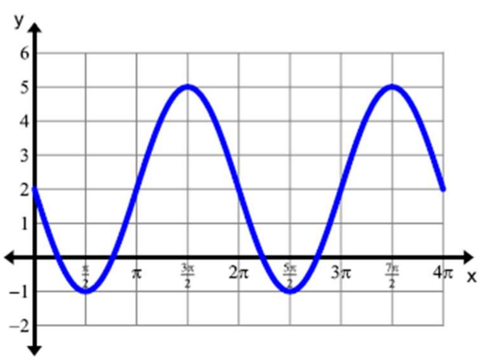
$$f(x) = a\sin(bx) + k \qquad \text{and} \qquad f(x) = a\cos(bx) + k$$

*****Pay attention to reflections*****

- **From 0, -sin(x) starts at midline, then goes down to minimum**
- **From 0, -cos(x) starts at minimum, then goes up**

Example 2:
Identify the amplitude and period, and determine where the midline is located. Then write the equation for the function.

a. Use $f(x) = \sin x$ for your parent graph.



b. Use $f(x) = \cos x$ for your parent graph.