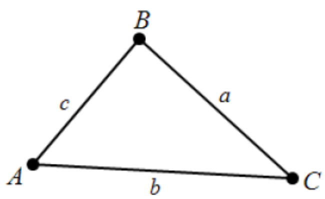
Math 3 - Law of Sines Notes

*****YOUR CALCULATOR MUST BE IN DEGEREE MODE*****

Law of Sines

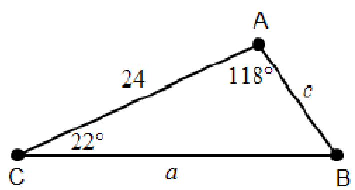
For any $\triangle ABC$, the Law of Sines relates the sine of each angle to the length of the side opposite the angle.



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

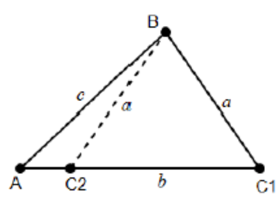
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Example 1:
Use the Law of Sines to solve the triangle. Round your answers to three decimal places.



The Ambiguous Case (SSA)

If you are given two angles and one side (ASA or AAS), the Law of Sines will easily provide ONE solution for a missing side. However, the Law of Sines has a problem dealing with SSA. If you are given two sides and one angle, where you must find an angle, the Law of Sines could possibly provide you with one or more solutions or even no solution at all.



Now check to see if more than one triangle exists with the given information.

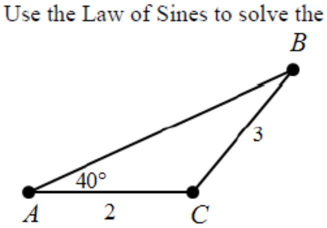
TEST

180°
- (the found angle)

Answer
+ (given angle)

If sum > 180° there is only one triangle
If sum < 180° there are two triangles

Example 2:
Use the Law of Sines to solve the triangle.



Example 3: Use the Law of Sines to solve the triangle

$C = 36°,\ a = 17,\ c = 16$

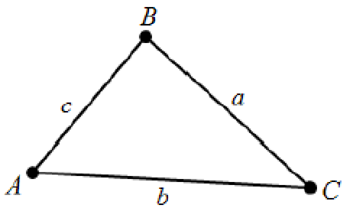
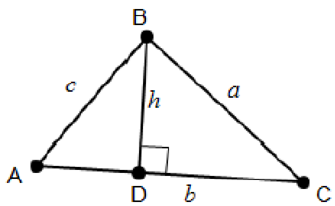
***When two triangles exist, the angle of the second triangle is 180-first angle found

Example 4: Use the Law of Sines to solve the triangle

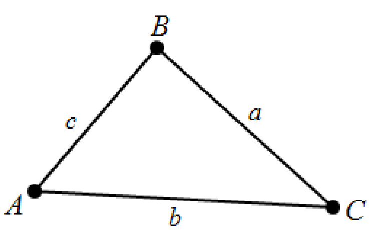
$A = 150°,\ a = 9.3,\ b = 41$

Proof of Law of Sines

Proof:

	Start with any triangle that has angles A, B, and C and side lengths, *a*, *b*, and *c*, where *a* is the side opposite angle A, *b* is the side opposite angle B, and *c* is the side opposite angle C.
	Construct an altitude, *h*, from one of the angles to the side opposite the angle. The two triangles formed, $\triangle ABD$ and $\triangle BDC$, are right triangles.
$\sin A = \frac{h}{c}$ and $\sin C = \frac{h}{a}$	Use the definition of sine to relate the base angles, $\angle A$ and $\angle C$, to the hypotenuse of each and the altitude.
$c \sin A = h$ and $a \sin C = h$	Use the multiplication property of equality to solve each equation for *h*.
$c \sin A = a \sin C$	Use the transitive property of equality to set the equations equal to one another.
$\frac{\sin A}{a} = \frac{\sin C}{c}$	Use the division property of equality.

You can also derive the Law of Sines from the formula for the area of a triangle given two sides and the included angle.

	All three of these formulas will give the same area for the triangle. $\text{area} = \frac{1}{2}bc(\sin A)$ $\text{area} = \frac{1}{2}ac(\sin B)$ $\text{area} = \frac{1}{2}ab(\sin C)$
$\frac{1}{2}bc(\sin A) = \frac{1}{2}ac(\sin B)$	Set the right side of the first two equations equal to one another.
$bc(\sin A) = ac(\sin B)$ $b(\sin A) = a(\sin B)$	Multiply each side of the equation by 2. Divide each side of the equation by *c*.
$\frac{\sin A}{a} = \frac{\sin B}{b}$	Divide each side by *ab*.