Secondary Math 3 - Area of a Triangle Notes

It is possible to find the area of a triangle using trigonometry when given two sides and the included angle. In order to do this, you must draw an altitude from the vertex of the non-included angle to the side opposite the angle.

Derivation of the area of a triangle

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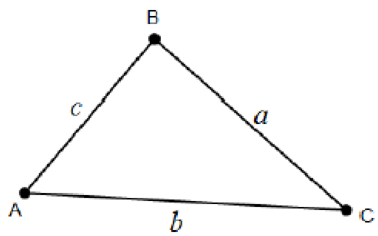
<td>
</td>
<td>Start with any triangle that has angles A, B, and C and side lengths, a, b, and c, where a is the side opposite angle A, b is the side opposite angle B, and c is the side opposite angle C.</td>
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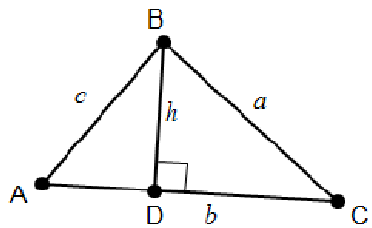
<td>
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<td>Construct an altitude, h, from the vertex of one of the angles to the side opposite the angle. The two triangles formed, $\triangle ABD$ and $\triangle BDC$, are right triangles.</td>
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<td>$\sin A = \frac{h}{c}$
$c \sin A = h$</td>
<td>Find the measure of h in terms of $\angle A$ and side c using the sine ratio for $\angle A$.</td>
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<tr>
<td>$\text{Area} = \frac{1}{2}(\text{base})(\text{height})$
$\text{Area} = \frac{1}{2}b(c \sin A)$
$\text{Area} = \frac{1}{2}bc(\sin A)$</td>
<td>The base of the entire triangle is b and the height is h. Substitute $c \sin A = h$ into the area formula for h, the height.

NOTE: This formula works for acute, obtuse, and right triangles.</td>
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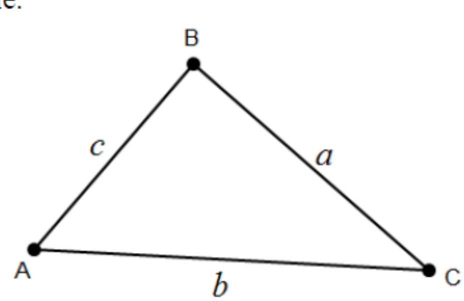
Area of a Triangle Given Two Sides and the Included Angle

The area of a triangle is half the product of the lengths of two sides and the sine of the included angle.



$$\text{Area} = \frac{1}{2}bc(\sin A)$$

$$\text{Area} = \frac{1}{2}ac(\sin B)$$

$$\text{Area} = \frac{1}{2}ab(\sin C)$$

YOUR CALCULATOR MUST BE IN DEGREE MODE

Examples:

1. Find the area of a triangle with sides $a = 11$, $b = 5$, and $m\angle C = 20°$. Round your answer to the nearest thousandth (3 decimal places).

2. Find the area of a triangle with sides $b = 13$, $c = 7$, and $m\angle A = 43°$. Round your answer to the nearest thousandth (3 decimal places).